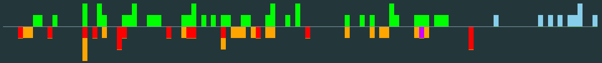
A: Accidental Arithmetic

Problem author: Freek Henstra



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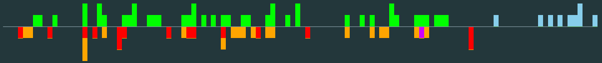

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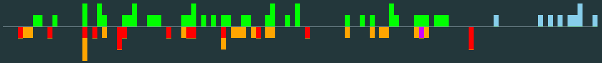

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Solution: The first term has k digits $(1 \leq k \leq d-1)$ with probability $0.1^{k-1} \cdot 0.9$, and d digits with probability 0.1^{d-1}. Multiplying by 0.1 just moves the decimal, so if, for example, $n = 1234$, the answer is $0.9 \cdot (1 + 1.2 + 1.23) + 1.234 = 4.321$.

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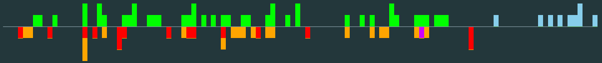

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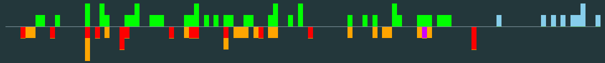

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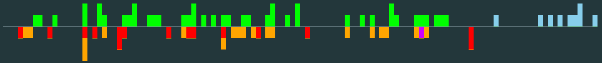

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Statistics: 87 submissions, 43 accepted, 9 unknown