## **D:** Duo Detection

Problem author: Mike de Vries

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  - When n is small, works well, but can be quadratic.
- **Naive solution** 2: Loop over all pairs of symbols in each message. Check if any of these pairs is the same, by using a hash map. Time complexity:  $\mathcal{O}(\sum_{i=1}^{n} |M_i|^2)$  Works well when the sizes of the messages are small.

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**Big-Big** Use naive solution 1. Works in  $\mathcal{O}(\sum_{i \in \mathsf{Bigs}} |M_i| \frac{m}{B}) = \mathcal{O}(\frac{m^2}{B})$ 

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- **Small-Small** Use naive solution 2, works in  $\mathcal{O}(mB)$  (worst case is all small messages are equal in size and < B.)

This handles all the cases, and total complexity is  $\mathcal{O}(\frac{m^2}{B} + mB)$ , best when  $B = \Theta(\sqrt{m})$ .

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**Running time:**  $\mathcal{O}(m\sqrt{m})$  with a hash map. The algorithm has a considerable constant factor.

**Bonus:** You can get rid of the hash map. This requires in naive solution 2 to order the computations in a smart way, such that a global boolean array can be used, which is set and unset for each message. Also requires sorting and coordinate compression beforehand. Same tricks need to be used in naive solution 1.

**Bonus** 2: The algorithm can be sped up to  $\mathcal{O}(m\sqrt{\frac{m}{w}})$ , where w is the word size of the machine (typically 32 or 64). This can be done with the use of bitsets in naive algorithm 1. The details are an exercise for the interested reader.

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Statistics: 98 submissions, 9 accepted, 40 unknown