## **E**: Excruciating Elevators

Problem author: Albert Eikelenboom

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Statistics: 16 submissions, 1 accepted, 7 unknown