F: Faulty Connection

Problem author: Mike de Vries

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Idea: Any two points on the plane define a unique line, and any two lines have at most one intersection point. We can assign lines to messages, and points on those lines to numbers. Since two different lines have at most one intersection point, two messages have at most one number in common.

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Solution: Use the lines in $\mathbb{F}_{31}^2$.

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In English: Take a 31×31 grid and perform all arithmetic modulo 31 (make it wrap around the boundaries like in the game of Asteroids). Use lines of the form $y = ax + b$ (with $a, b \in \{0, 1, ..., 30\}$).

Assign a line to each message, and a number to each point.

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Bonus: It is possible to solve the problem for $31^2 + 31 + 1 = 993$ messages of 32 numbers using only numbers up to 993.

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Example: One message may correspond to the red line, and another to blue. Receiving the points $(2, 28)$ and $(16, 8)$, the red line can be reconstructed using modular arithmetic. The red and blue line ($y = 1x + 5$) intersect in exactly one integer point: $(7, 12)$.

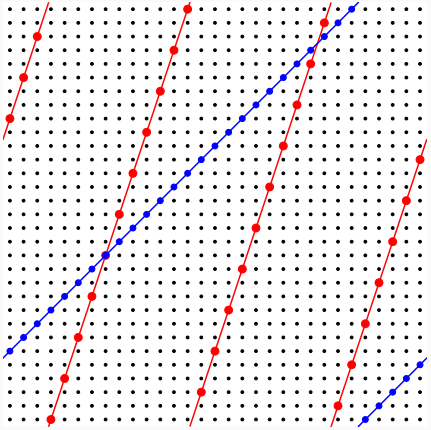

Figure 1: Lines corresponding to $y = 3x + 22$ (red) and $y = 1x + 5$ (blue).

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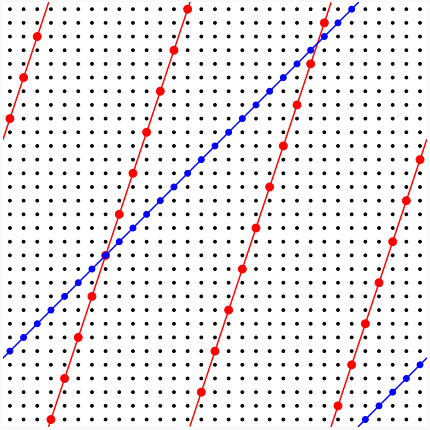

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Statistics: 42 submissions, 10 accepted, 24 unknown