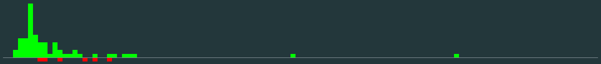
J: Jacobi Numbers

Problem author: Reinier Schmiermann



Problem: Write n as a sum of cubes.

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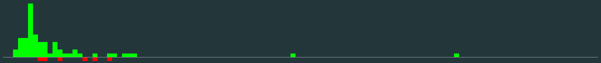

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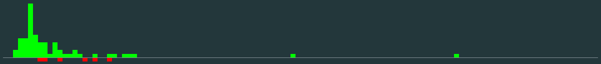

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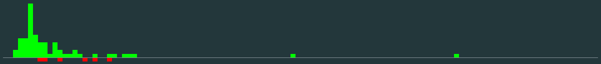

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Decompose: Decompositions of the numbers between 1 and 9241 into sums of cubes using as few terms as possible:

singular cubes,	like $27 = 3^3$	20
sums of two cubes,	like $9241 = 56^3 + (-55)^3$	453
sums of three cubes,	like $9240 = 56^3 + (-55)^3 + (-1)^3$	5761
sums of four cubes,	like $9239 = 32^3 + (-25)^3 + (-22)^3 + 14^3$	3007

Note that the terms can be much larger than the sum, *e.g.*,

$$311 = -9529^3 - 8185^3 + 8228^3 + 9497^3 .$$

Careful C++ implementation computes this within time bounds.

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Constant time: Determine (optimal) decomposition off-line for all possible inputs; the submission then looks up input in table with 10 000 entries.

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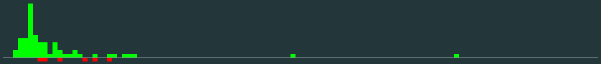

Known: Every integer is the sum of five cubes.

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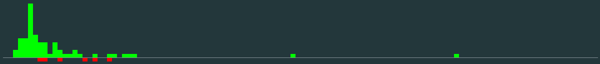

Known: Every integer is the sum of five cubes.

Proof: When $n = 6r$ for integer r we have

$$n = (r+1)^3 + (r-1)^3 + 2(-r)^3 .$$

Possibly adding $(-1)^3$ on the right hand side solves the problem for $6r, 6r-1, 6r+1$. The remaining cases $(6r+2, 6r+3, 6r+4)$ are handled similarly. There are many ways of doing this.

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Open problem: Can every integer be decomposed into four cubes?

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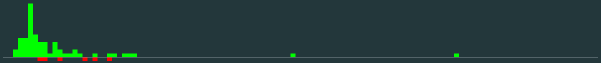

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Statistics: 66 submissions, 60 accepted