

Problem Analysis Session

EUC 2025 judges

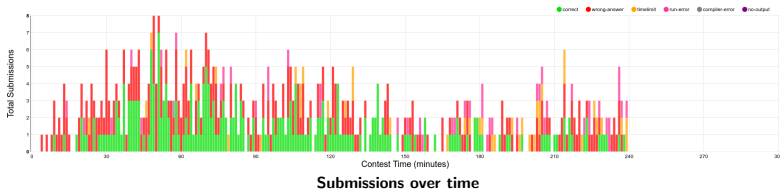
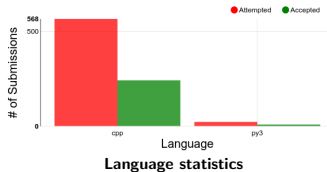
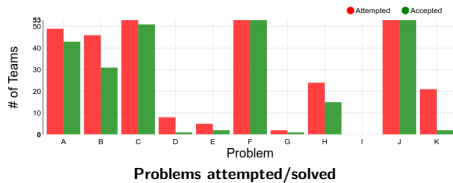
March 2, 2025

Our judges and problemsetters

- Federico Glaudo (Chief judge)
- Lucian Bicsi
- Andrea Ciprietti
- Jorke De Vlas
- Tomaz Hocevar
- Egor Kulikov
- Petr Mitrichev
- Pedro Paredes
- Giovanni Paolini
- Alice Sayutina

Statistics (after freeze)

Total number of **submissions**: 591
of which **accepted**: 252 ($\sim 42\%$)



Number of **submissions**: 76
of which **accepted**: 53



First solved by **Union University - Faculty of Computer Science**
after 20m





Mascot Naming

AUTHORED BY: Federico Glaudo

PREPARED BY: Federico Glaudo

The problem

You are given n strings $s_1, s_2, \dots, s_n$, and a string t . Find (if it exists) a string that has $s_1, \dots, s_n$ as subsequences but does not have t as a subsequence.



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Solution

- If t is a subsequence of one s_i , then the answer is NO

The problem

You are given n strings $s_1, s_2, \dots, s_n$, and a string t . Find (if it exists) a string that has $s_1, \dots, s_n$ as subsequences but does not have t as a subsequence.

Solution

- If t is a subsequence of one s_i , then the answer is NO
- Otherwise, construct the answer in a **greedy** fashion: remove the first character from one of the s_i and append it to the answer

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Example

$s_1 = \text{abc}$, $s_2 = \text{caba}$, $s_3 = \text{aacbb}$

$t = \text{cbc}$

$ans =$



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$t = \text{cbc}$

$ans = \quad \leftarrow a$

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$t = \text{cbc}$

$\text{ans} = \text{a}$

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Example

$s_1 = \text{ab}\textcolor{red}{c}$, $s_2 = \textcolor{red}{c}\text{aba}$, $s_3 = \text{a}\textcolor{red}{a}\text{cbb}$

$t = \textcolor{red}{c}\text{bc}$

$\textit{ans} = \text{ab}$

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$t = \textcolor{red}{c}\text{bc}$

$\text{ans} = \text{ab} \quad \leftarrow \text{a}$



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Example

$s_1 = \text{ab}\textcolor{red}{c}$, $s_2 = \textcolor{red}{c}\text{aba}$, $s_3 = \text{aa}\textcolor{red}{c}\text{bb}$

$t = \textcolor{red}{c}\text{bc}$

$\textit{ans} = \text{aba}$



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$t = \textcolor{red}{c}\text{bc}$

$\text{ans} = \text{aba} \quad \leftarrow \textcolor{red}{c}$



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Example

$s_1 = abc$, $s_2 = caba$, $s_3 = aacbb$

$t = cb$

$ans = abac$



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Example

$s_1 = abc$, $s_2 = ca$ **ba**, $s_3 = aa$ **cbb**

$t = c$ **b** c

$ans = abaca$



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$t = c$ **b** c

$ans = abaca \quad \leftarrow b$



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$t = cb$ **c**

$ans = abacab$



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$t = cb$ **c**

$ans = abacab \quad \leftarrow a$



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$t = cb$ **c**

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$t = \text{cb}\textcolor{red}{c}$

$\text{ans} = \text{abacabab}$

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- **Achtung!** The most naive implementation of this algorithm has time complexity $n \cdot (|s_1| + \dots + |s_n| + |t|)$, which is too slow

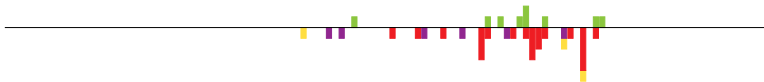
J The Ultimate Wine Tasting Event

AUTHORED BY: Andrea Ciprietti PREPARED BY: Andrea Ciprietti

Number of **submissions**: 115
of which **accepted**: 53



First solved by **Masaryk University** after 13m



J The Ultimate Wine Tasting Event

AUTHORED BY: Andrea Ciprietti PREPARED BY: Andrea Ciprietti

The problem

Given a sequence of $2n$ wines, n white and n red, can you split them into two subsequences of size n , swap them element-wise, and end up with all white wines on the left?

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Solution

- Find an **equivalent condition** that is easy to verify

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Solution

- Find an **equivalent condition** that is easy to verify
- Assuming it is possible, denote the two subsequences $a_1 < \dots < a_n$ and $b_1 < \dots < b_n$

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- Let's pick h and k such that:

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- Assuming it is possible, denote the two subsequences $a_1 < \dots < a_n$ and $b_1 < \dots < b_n$
- Let's pick h and k such that:
 - a_h and b_k are in the first half;

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- Let's pick h and k such that:
 - a_h and b_k are in the first half;
 - a_{h+1} and b_{h+1} are in the second half (or either h or k is n);

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 - $h \leq k$ (WLOG)

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- Assuming it is possible, denote the two subsequences $a_1 < \dots < a_n$ and $b_1 < \dots < b_n$
- Let's pick h and k such that:
 - a_h and b_k are in the first half;
 - a_{h+1} and b_{h+1} are in the second half (or either h or k is n);
 - $h \leq k$ (WLOG)
- If $\{a\}$ has no bottles in the first half, set h to 0

J The Ultimate Wine Tasting Event

AUTHORED BY: Andrea Ciprietti PREPARED BY: Andrea Ciprietti

Solution

- After swapping:

J The Ultimate Wine Tasting Event

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Solution

- After swapping:
 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;

J The Ultimate Wine Tasting Event

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Solution

- After swapping:
 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;
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- After swapping:
 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;
 - bottles $b_{h+1}, \dots, b_k$ end up in the second half, so they are red
- $\implies$ There are exactly $2h$ white wines in the first half

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 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;
 - bottles $b_{h+1}, \dots, b_k$ end up in the second half, so they are red
- $\implies$ There are exactly $2h$ white wines in the first half
- Also, the first h wines are white

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Solution

- After swapping:
 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;
 - bottles $b_{h+1}, \dots, b_k$ end up in the second half, so they are red
- $\implies$ There are exactly $2h$ white wines in the first half
- Also, the first h wines are white
- By symmetry, the second half has $2h$ red wines and the last h are red

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Solution

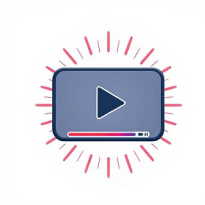
- After swapping:
 - bottles $a_1, \dots, a_h$ and $b_1, \dots, b_h$ end up in the first half, so they are white;
 - bottles $b_{h+1}, \dots, b_k$ end up in the second half, so they are red
- $\implies$ There are exactly $2h$ white wines in the first half
- Also, the first h wines are white
- By symmetry, the second half has $2h$ red wines and the last h are red
- These conditions are sufficient! **Exercise!** :)



Ads

AUTHORED BY: Andrea Ciprietti PREPARED BY: Tomaz Hocevar

Number of **submissions**: 101
of which **accepted**: 51



First solved by **Ecole Polytechnique Fédérale de Lausanne** after 36m





Ads

AUTHORED BY: Andrea Ciprietti PREPARED BY: Tomaz Hocevar

The problem

Minimize the number of shown ads by rearranging n videos of lengths d_i , where ads are shown between videos after every 3 videos or after $\geq k$ minutes of content.



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Solution

- Ads partition the videos into groups $\rightarrow$ Minimize the number of groups

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Solution

- Ads partition the videos into groups $\rightarrow$ Minimize the number of groups
- All *long* videos (length $\geq k$) go in different groups



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- All *long* videos (length $\geq k$) go in different groups
- Make as many groups of 3 videos as possible by finding pairs of *short* videos with total length $< k$



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Solution

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- All *long* videos (length $\geq k$) go in different groups
- Make as many groups of 3 videos as possible by finding pairs of *short* videos with total length $< k$
- Pair up the remaining *short* videos with the remaining *long* videos, and any leftover *short* videos form groups of size 1



The problem

Minimize the number of shown ads by rearranging n videos of lengths d_i , where ads are shown between videos after every 3 videos or after $\geq k$ minutes of content.

Solution

- Ads partition the videos into groups $\rightarrow$ Minimize the number of groups
- All *long* videos (length $\geq k$) go in different groups
- Make as many groups of 3 videos as possible by finding pairs of *short* videos with total length $< k$
- Pair up the remaining *short* videos with the remaining *long* videos, and any leftover *short* videos form groups of size 1
- **Complexity:** $O(n \log n)$ for sorting, then linear

Number of **submissions**: 67
of which **accepted**: 43



First solved by **Ecole Polytechnique** after 9m



The problem

There is an election where each vote is a ranked list of candidates.
You are given n claims:

- “Candidate a_i is preferred to b_i by more than half voters”

Construct a valid list of votes.

The problem

There is an election where each vote is a ranked list of candidates. You are given n claims:

- “Candidate a_i is preferred to b_i by more than half voters”

Construct a valid list of votes.

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- We need to find ans: $\delta_{a,b}(\text{ans}) > 0$ for all required (a, b)

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$\delta_{a,b}(\text{voters})$ is how many votes prefer a to b minus how many votes prefer b to a

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- This uses at most $2\binom{n}{2} \leq n^2$ voters



Urban Planning

AUTHORED BY: Petr Mitrichev PREPARED BY: Petr Mitrichev

Number of **submissions**: 114
of which **accepted**: 31



First solved by **ETH Zürich** after 40m



B Urban Planning

AUTHORED BY: Petr Mitrichev PREPARED BY: Petr Mitrichev

The problem

Generate a grid of size at most 2025×2025 with # and . characters which has exactly k rectangles of #, where $k \leq 4 \cdot 10^{12}$ is given

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- Still, there are many approaches that work, not a single "correct" solution. Here is one approach
- Let us have several connected components. First, we try to have a large one that has close to k rectangles, and then use additional components to cover the difference



Urban Planning

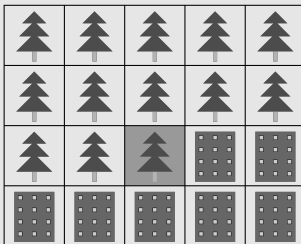
AUTHORED BY: Petr Mitrichev PREPARED BY: Petr Mitrichev

Solution

- We start with a 1×2025 rectangle, and then add additional rows. In each row, we add cells one by one, while we still have less than k rectangles

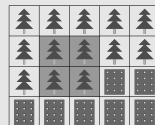
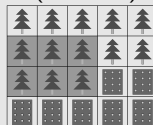
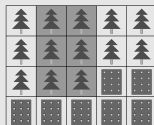
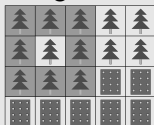
Solution

- We start with a 1×2025 rectangle, and then add additional rows. In each row, we add cells one by one, while we still have less than k rectangles
- (We replace 2025 with 5 for the purpose of the picture, and the last added cell is highlighted)



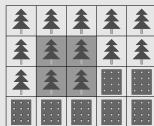
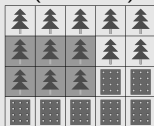
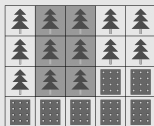
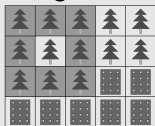
Solution

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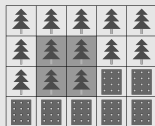
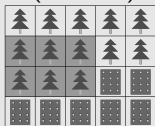
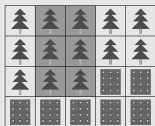
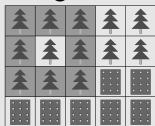
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- We reach $4 \cdot 10^{12}$ rectangles within 1977 rows, so we can stop right before that and we will be at most $1976 \cdot 2024 - 1 = 3\,999\,423$ rectangles short of the required k

Solution

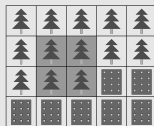
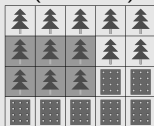
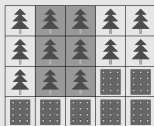
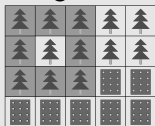
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- Now we can skip one row, and start solving the same problem but with $k \leq 3\,999\,423$
- We will have one more block with at most 3 rows, and at most six more blocks with two rows, and we're done!

B Urban Planning

AUTHORED BY: Petr Mitrichev PREPARED BY: Petr Mitrichev

Additional thoughts

- The online mirror has tougher constraints $k \leq 4.194 \cdot 10^{12}$



Urban Planning

AUTHORED BY: Petr Mitrichev PREPARED BY: Petr Mitrichev

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- **Hint:** Consider a 2023×2024 rectangle. What happens if we add some cells to the right of it, and some cells to the bottom of it? What numbers can be represented as $2023x + 2024y$?
- For the judges' solution, it does not matter if the rectangles are hollow or not. We made them hollow so that counting rectangles takes $\mathcal{O}(n^2 \log n)$ instead of $\mathcal{O}(n^2)$, so hopefully simple hill-climbing solutions were less likely to work

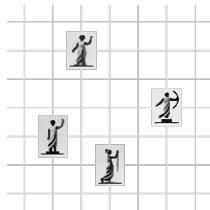


Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini

PREPARED BY: Giovanni Paolini

Number of **submissions**: 61
of which **accepted**: 15



First solved by **ETH Zürich** after 1h25m





Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini PREPARED BY: Giovanni Paolini

The problem

Given the location of the first and last statue in a 2D city, arrange the remaining $n - 2$ statues so that the Manhattan distance between the i -th and the $(i + 1)$ -th statue is exactly d_i (if possible).

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AUTHORED BY: Federico Glaudo & Giovanni Paolini PREPARED BY: Giovanni Paolini

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- Let d_0 be the distance between the first and last statue
- **Key insight:** An arrangement exists if and only if
 - (1) $d_0 + d_1 + \cdots + d_{n-1}$ is even;
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- The two conditions are necessary for $d_0, d_1, \dots, d_{n-1}$ to form the (Manhattan) side lengths of a polygon. In particular, (2) follows from the triangle inequality

Solution

- Conversely, assume that the two conditions hold:

(1) $d_0 + d_1 + \cdots + d_{n-1}$ is even;

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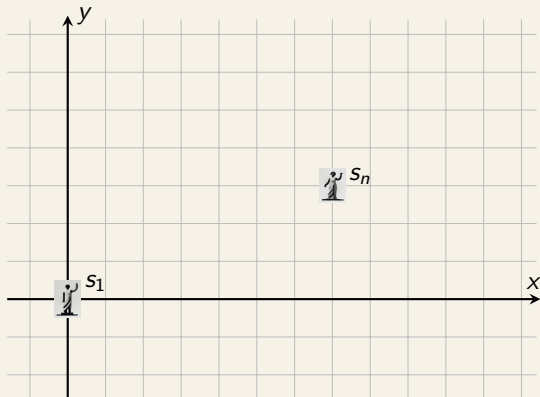
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 - $\text{distance}(s_{n-1}, s_n) = d_{n-1}$;
 - if we define $\hat{d}_0 = \text{distance}(s_1, s_{n-1})$, then the distances $\hat{d}_0, d_1, \dots, d_{n-2}$ satisfy (1) and (2)



Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini PREPARED BY: Giovanni Paolini

Solution

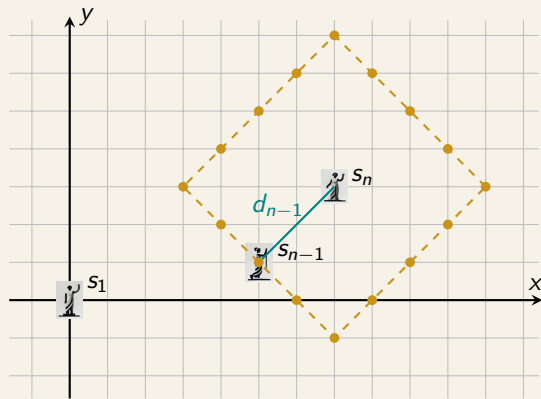


H Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini

PREPARED BY: Giovanni Paolini

Solution



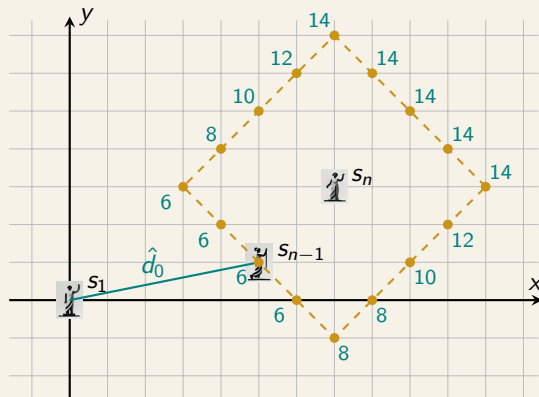
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H Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini

PREPARED BY: Giovanni Paolini

Solution



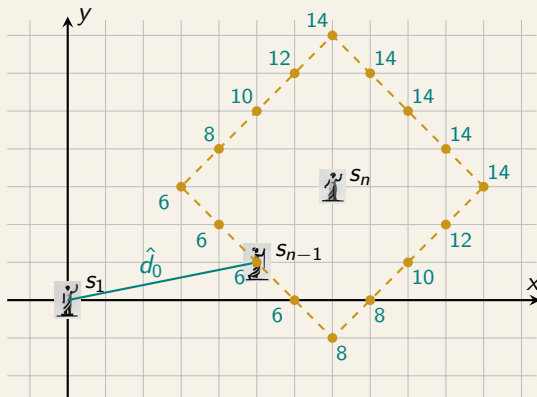
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H Statues

AUTHORED BY: Federico Glaudo & Giovanni Paolini

PREPARED BY: Giovanni Paolini

Solution



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- The parity of $\hat{d}_0$ ensures that $\hat{d}_0, d_1, \dots, d_{n-2}$ satisfy condition (1)

Solution

- **Claim:** The value $\hat{d}_0 := \min(d_0 + d_{n-1}, d_1 + \dots + d_{n-2})$ belongs to the admissible range $\{|d_0 - d_{n-1}|, \dots, d_0 + d_{n-1}\}$, has the correct parity, and makes $\hat{d}_0, d_1, \dots, d_{n-2}$ satisfy condition (2)

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- A suitable location for s_{n-1} can then be found in $\mathcal{O}(1)$ time
- Overall, we have an $\mathcal{O}(n)$ algorithm to construct a valid arrangement if conditions (1) and (2) are initially satisfied



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

Number of submissions: 7
of which **accepted**: 2



First solved by **Scuola Normale Superiore** after 2h21m





Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

The problem

A graph is given and an agent (supporters' club) needs to go from node 1 to node n . An adversary (police), that knows the agent movements, can, at any time and exactly once, remove an edge.

What is the minimum time required to the agent to reach n , if they play optimally?



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Solution

- Let's call $f(v)$ the minimum number of roads needed to reach n from v
- So $f(1)$ is the answer and $f(n)$ is 0



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

Solution

- Suppose no road has been blocked and supporters are in v



Porto Vs. Benfica

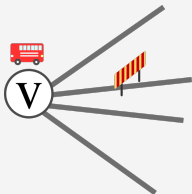
AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

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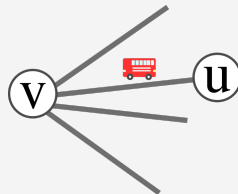
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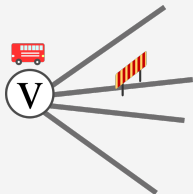
Block some road adjacent to v



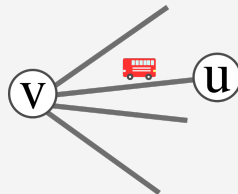
Let the supporters pick some road

Solution

- Suppose no road has been blocked and supporters are in v
- Then the police does one of 2 things:



Cost is $g(v, e) \rightarrow g(v)$



Cost is $f(u) \rightarrow 1 + \min_u(f(u))$

- Let $g(v, e)$ be the shortest path from v to n with edge e blocked, and let $g(v) := \max(g(v, e))$ for adjacent e



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

Solution

- **Observation 1:** $f(v) = \max(g(v), 1 + \min_{u \sim v}(f(u)))$



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

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- So, now we need to compute g and then f
- Let's first find f assuming we know g



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

Solution: Computing f assuming g

- We can compute $f(v) = \max(g(v), 1 + \min_{u \sim v}(f(u)))$ using a greedy Dijkstra-like algorithm



Porto Vs. Benfica

AUTHORED BY: Pedro Paredes PREPARED BY: Pedro Paredes

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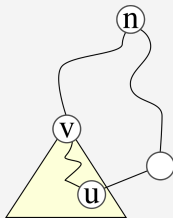
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- **Observation 2:** At the end $f(v) = \mathfrak{f}[v]$

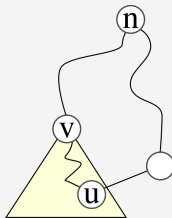
Solution: Computing g

- Consider the BFS tree from n



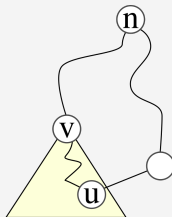
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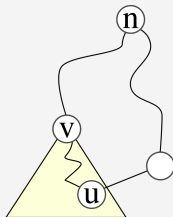
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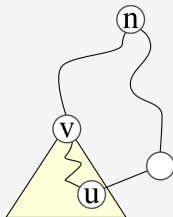
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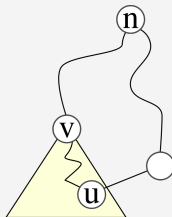
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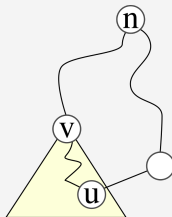
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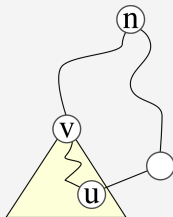
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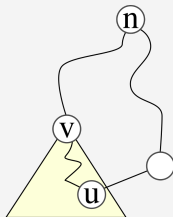
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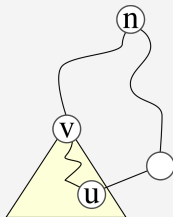
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- Find all valid paths from each v
- Recursively compute distances for all children of v
- Initialize sets by adding pairs $(1 + \text{dist}(u), u)$ for each non-tree edge



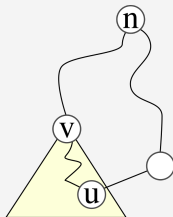
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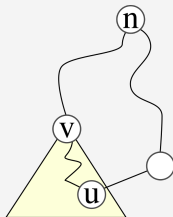
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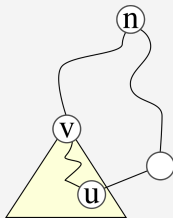
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- The remaining minimum element in the set gives $g(v)$



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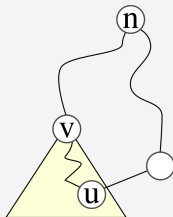


Porto Vs. Benfica

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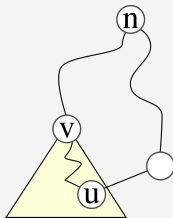
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- Use small-to-large merging to efficiently merge sets



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- Use DSU (union-find) to check if u is in v 's subtree
- Use small-to-large merging to efficiently merge sets
- Total running time of this step is $\mathcal{O}(n \log^2 m)$



Number of submissions: 33
of which **accepted**: 2



First solved by **Jagiellonian University in Krakow** after 2h27m



The problem

We are given a sequence of n numbers a_i . We need to find another sequence of n *distinct* numbers b_i such that:

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(Output only the minimum)

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- **Strategy:** Build a graph and run the max flow algorithm
- When we reach flow n , the last vertex added will determine the answer
- Denote $p_1, \dots, p_k$ the distinct values that appear in the sequence, where p_j appears q_j times

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- **Vertices:** The initial set of vertices is made up of:
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- Continue adding vertices B_i in increasing order of i until the (S, T) max flow reaches n . The answer to the problem is the last index i added

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- Initially, set $M(p_j) = \{j\}$ for $1 \leq j \leq k$, and $M(i) = \emptyset$ otherwise

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 - It can be shown that $E = \mathcal{O}(n \log n)$, making the method efficient

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- We construct a flow graph with vertices $\{A_i\}$ and $\{B_i\}$ to model the problem

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- An optimized approach uses a map $M(i)$ to only process necessary indices, reducing the number of steps
- The final complexity is $\mathcal{O}(n \cdot E)$ with $E = \mathcal{O}(n \log n)$



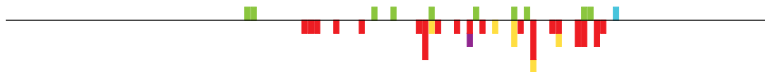
Morse Code

AUTHORED BY: Jorke De Vlas PREPARED BY: Jorke De Vlas

Number of **submissions**: 10
of which **accepted**: 1

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First solved by **Karlsruhe Institute of Technology** after 3h31m





Morse Code

AUTHORED BY: Jorke De Vlas PREPARED BY: Jorke De Vlas

The problem

Given a set of character frequencies, determine a Morse Code like assignment where no codes are prefixes of each other and where the expected transmission time per character is minimal.

A	$f_A = 0.3$
B	$f_B = 0.6$
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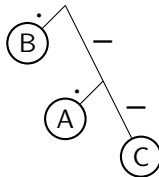
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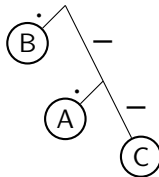
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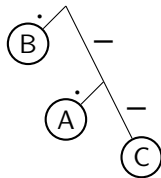
Solution

- We can describe such an assignment using a binary tree
- Only place characters on leaves to avoid prefixes

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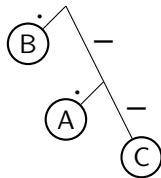
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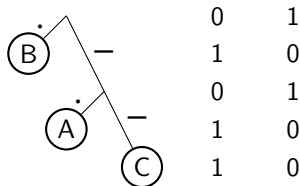
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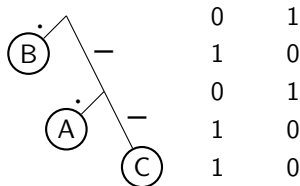
Solution

- If we know the shape of the tree, we can assign characters greedily
- To determine the shape, we only need to know the number of leaves and internal vertices per level



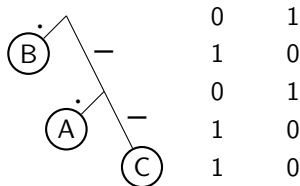
Solution

- The optimal vertex sequences can be determined by trying all possible suffixes with **Dynamic Programming**



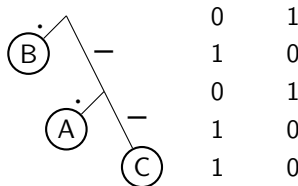
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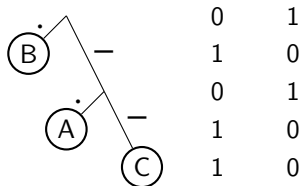
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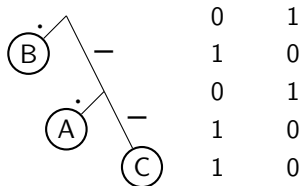
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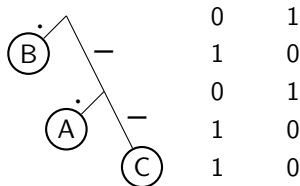
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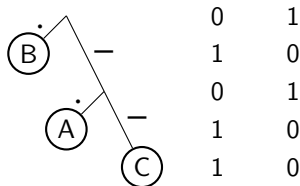
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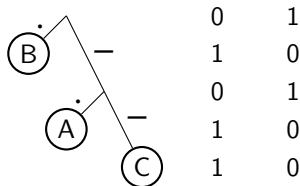
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- The value $\text{dp}(v_c, v_n, a)$ describes the expected cost of all characters below the current level
- **Sought value:** $\text{dp}(1, 0, n)$
- **Base case:** $\text{dp}(0, 0, 0) = 0$



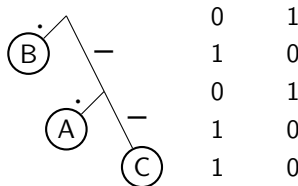
Solution

- The value $\text{dp}(v_c, v_n, a)$ describes the expected cost of all characters below the current level
- **Sought value:** $\text{dp}(1, 0, n)$
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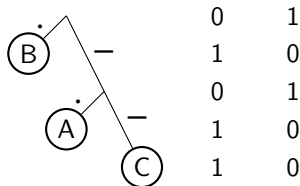
Morse Code

AUTHORED BY: Jorke De Vlas PREPARED BY: Jorke De Vlas



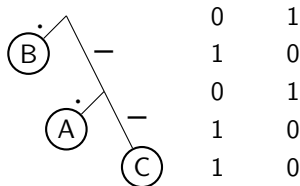
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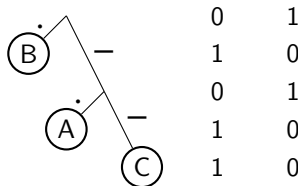
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 - We do not want more leaves, and go to the next level:
 $\text{dp}(v_c, v_n, a) = \text{dp}(v_n + v_c, v_c, a) + F(a)$, where $F(a)$ is the sum of the a smallest frequencies



Morse Code

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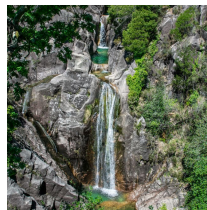
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- This gives a runtime of $\mathcal{O}(n^3)$
- To reconstruct the codes, use backtracking, keeping track of a set of partial codes at the current and the next level



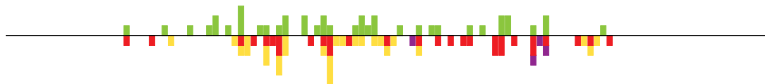
A Very Long Hike

AUTHORED BY: Federico Glaudo PREPARED BY: Federico Glaudo

Number of **submissions**: 6
of which **accepted**: 1



First solved by **Delft University of Technology** after 3h34m





A Very Long Hike

AUTHORED BY: Federico Glaudo PREPARED BY: Federico Glaudo

The problem

You are given an infinite grid where each cell contains a number and the $n \times n$ pattern of numbers is repeated periodically. You can move from a cell with number x to an adjacent cell with number y in $|x - y| + 1$ seconds. How many cells can you reach, approximately, in $R = 10^{20}$ seconds?



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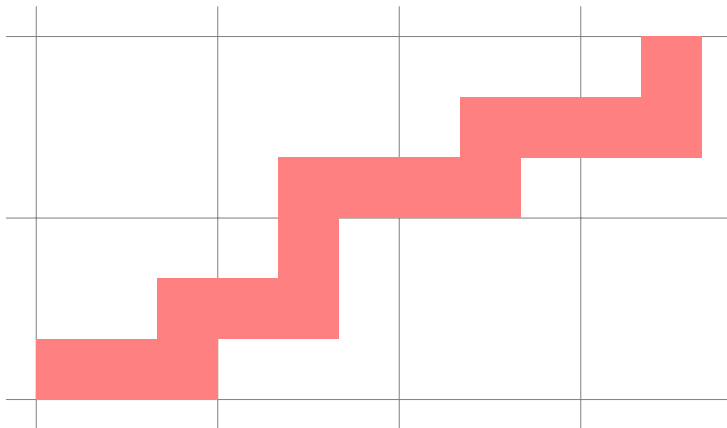
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A Very Long Hike

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$$n = 3$$

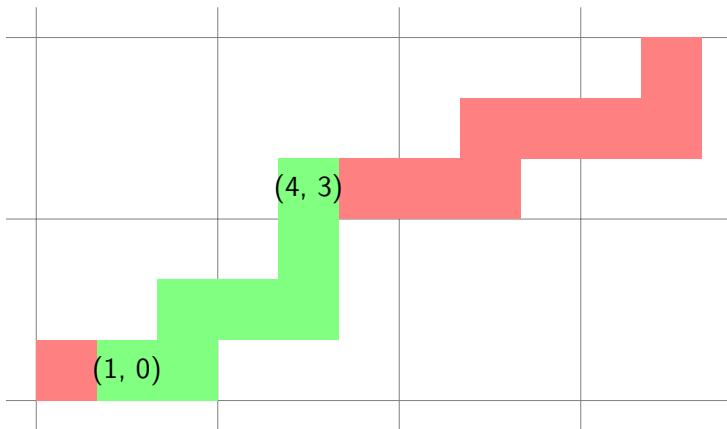




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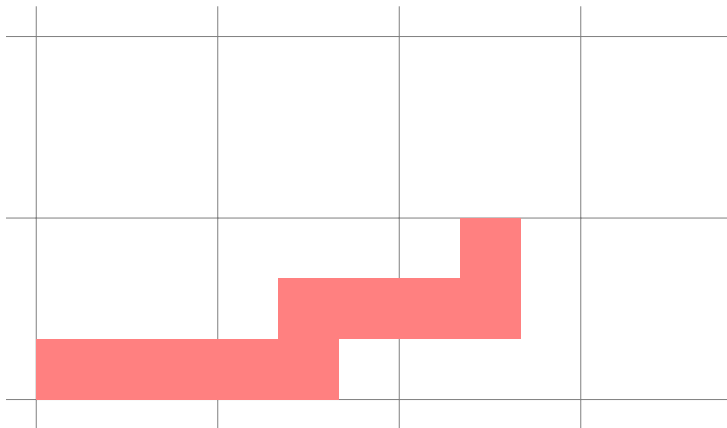




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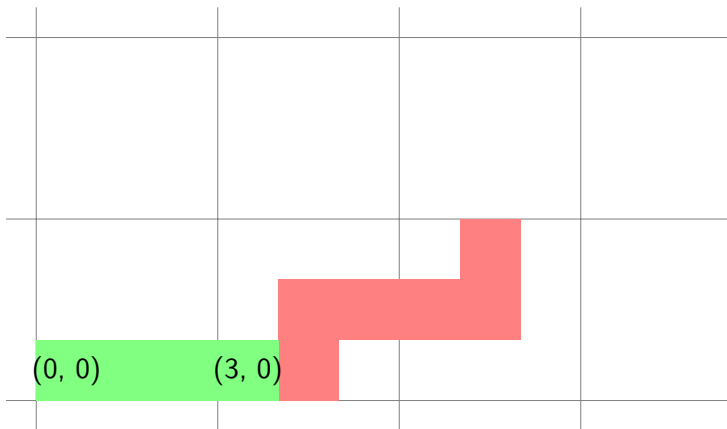




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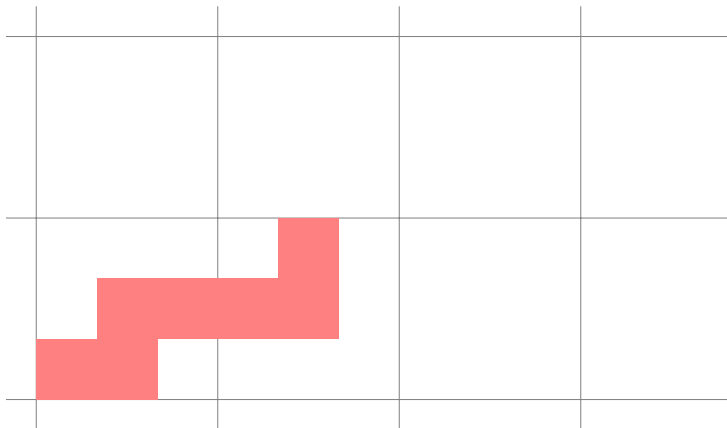




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- **Main idea (“converse”):** Any sum of interesting paths can be *obtained* from a given path up to adding at most $\text{poly}(n)$ cells



A Very Long Hike

AUTHORED BY: Federico Glaudo PREPARED BY: Federico Glaudo

Subproblem equivalent to the original one

For a given (a, b) , for any i, j with $|i| + |j| \leq n$, $T(i, j)$ is the minimum distance from (a, b) to $(a + in, b + jn)$ divided by n . We start from $(0, 0)$ and we can jump to (i, j) in $T(i, j)$ seconds. How many cells can we reach in $R = 10^{20}$ seconds?

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Solution: Solving the subproblem

If a, b are large,

$$\text{dist}((0, 0), (a, b)) \approx \inf \left\{ \sum_{|i|+|j| \leq n} \lambda_{ij} T(i, j) : \begin{array}{l} \lambda_{ij} \geq 0, \\ \sum_{|i|+|j|} \lambda_{ij} (i, j) = (x, y) \end{array} \right\}$$



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Solution: Solving the subproblem

The answer to the subproblem is

$$\text{area}(P) \cdot R^2,$$

where P is the convex hull of the points $p_{ij} := \left(\frac{i}{T(i,j)}, \frac{j}{T(i,j)} \right)$



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Solution: Computing the answer

- It remains only to compute $T(i, j)$ for all $|i| + |j| \leq n$



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- **The clever way:** Iterate only over the initial points (a, b) . The total runtime is $O(n^6 \log(n))$, which is enough!
- **The clever-er way:** Any path that begin and ends in cells equivalent modulo n passes through a cell with one of the two coordinates divisible by n . Iterate only over the initial points (a, b) with $a = 0$ or $b = 0$ and implement a linear-time Dijkstra. The total runtime is $O(n^5)$



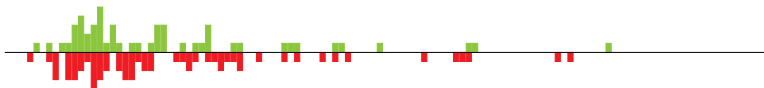
Pinball

AUTHORED BY: Lucian Bicsi PREPARED BY: Lucian Bicsi

Number of **submissions**: 0
of which **accepted**: 0



First solved by **N/A** after N/A





Pinball

AUTHORED BY: Lucian Bicsi PREPARED BY: Lucian Bicsi

The problem

You are playing a game on a $h \times w$ grid consisting of free cells, block-type walls, and oblique walls. Direct a ball towards exiting the grid by destroying oblique walls at precise moments in time.



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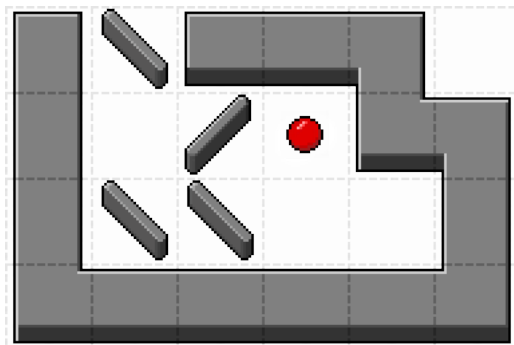
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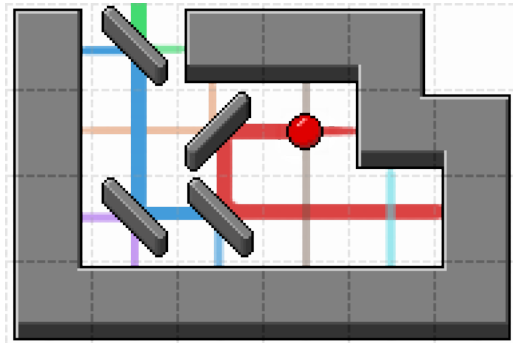
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- The movement of the ball can be grouped into (linear or circular) *regions*
- Destroying a walls allows the ball to transition from a region to another region

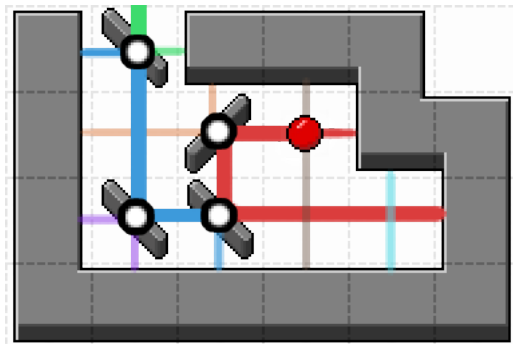


Solution



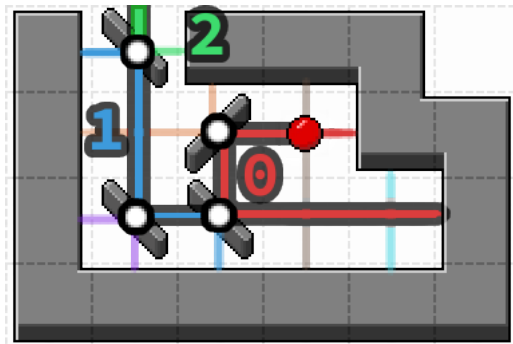
Solution

- Imagine a graph where each vertex is a region (red, blue, brown, cyan, purple, green)



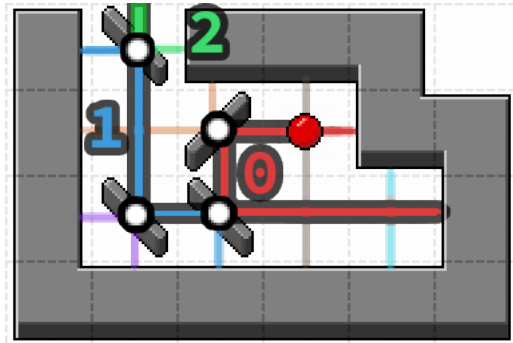
Solution

- Imagine a graph where each vertex is a region (red, blue, brown, cyan, purple, green)
- Add an edge between two regions if they share an oblique wall (red-blue, red-orange, blue-purple, blue-green)



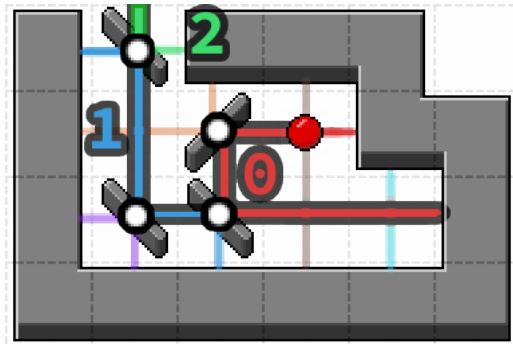
Solution

- Compute the minimum distance between any of the two starting (red, brown) regions and any of the regions that would allow the ball to escape (green)
- **Key fact!** The minimum distance is the answer to our problem!



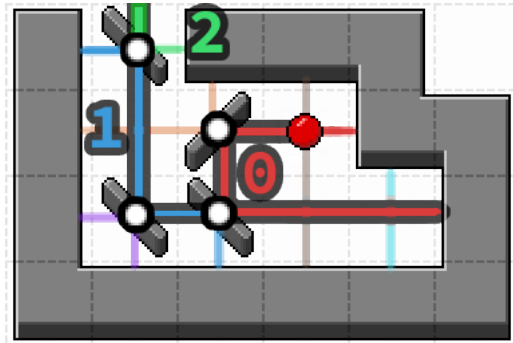
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- ... or we can just prove constructively!

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- **Achtung!** Careful implementation helps solving this problem significantly