Borders: The Problem

- Consider a black and white image, where every pixel has a value of either `0` or `1`.
- Define a region of the image as a collection of pixels that all have the same value, and are interconnected.
- You need to make sure every region has a border.
- What’s the minimum number of regions you must draw a border around to ensure that every region has a border?

0	0	0
1	1	1
0	0	0

= 3

0	0	0
0	1	0
0	0	0

= 1

0	1	0
1	0	1
0	1	0

= 8

Borders: Solution

- Firstly, every region on the edge must have its border drawn
  - There's no other way to get the outside edge borders
- For the inside, form a graph
  - Every region is a node
  - There's an edge between nodes if their regions need a border between them
- Now, the problem is Vertex Cover
  - i.e., minimum number of nodes to cover all edges
- Vertex Cover is NP-Complete in general, BUT this is a Bipartite Graph!
  - 0-region nodes, 1-region nodes, with only edges 0↔1, never 0↔0 or 1↔1
- Konig's Theorem says that you can find a Vertex Cover in a Bipartite Graph by finding a maximal matching
  - So, use your favorite matching technique!

Borders: Non-Solution 1

- There are several techniques that look like they might work, but won't.
- The first of these: After handling the edges, count the number of 0-regions, and the number of 1-regions, and use the smaller of the two.

0	0	1	0	1	1
0	1	0	1	0	1
1	0	0	1	1	0
0	1	0	1	0	1
0	0	1	0	1	1

- In this case, after handling edges, there are 3 0-regions and 3 1-regions, But, the interior can be handled by drawing borders around the two large T-shaped regions. 10 edge regions + 2 interior = 12

Borders: Non-Solution 2

- How about a Greedy algorithm?
- For the interior, always choose the region with the most edges first.

1	1	1	0	1	0	0	0	0	0
1	1	0	1	0	1	1	1	1	0
1	0	1	0	0	1	1	0	1	0
1	1	0	1	0	1	0	1	1	0
1	0	1	0	0	1	1	1	1	0
1	1	0	1	1	0	0	0	0	0

- In this case, the interior region with the most edges is the large 0-region.
- But, you still have to draw around all of the 1-regions to get all of the singleton 0-regions (or use the four leftmost 0-regions instead of the 4 singleton 1-regions).
- You can do the job by just drawing around the 1-regions.