

Nordic Collegiate Programming Contest

NCPC 2020

November 7, 2020



Problems

- A Array of Discord
- B Big Brother
- C Coin Stacks
- D Dams in Distress
- E Exhaustive Experiment
- F Film Critics
- G Gig Combinatorics
- H Hiring and Firing
- I Infection Estimation
- J Joining Flows
- K Keep Calm And Carry Off
- L Language Survey
- M Methodic Multiplication

Do not open before the contest has started.

Advice, hints, and general information

- The problems are **not** sorted by difficulty.
- Your solution programs must read input from *standard input* (e.g. `System.in` in Java or `cin` in C++) and write output to *standard output* (e.g. `System.out` in Java or `cout` in C++). For further details and examples, please refer to the documentation in the help pages for your favorite language on Kattis.
- For information about which compiler flags and versions are used, please refer to the documentation in the help pages for your favorite language on Kattis.
- Your submissions will be run multiple times, on several different inputs. If your submission is incorrect, the error message you get will be the error exhibited on the first input on which you failed. E.g., if your instance is prone to crash but also incorrect, your submission may be judged as either “Wrong Answer” or “Run Time Error”, depending on which is discovered first. The inputs for a problem will always be tested in the same order.
- If you think some problem is ambiguous or underspecified, you may ask the judges for a clarification request through the Kattis system. The most likely response is “No comment, read problem statement”, indicating that the answer can be deduced by carefully reading the problem statement or by checking the sample test cases given in the problem, or that the answer to the question is simply irrelevant to solving the problem.
- In general we are lenient with small formatting errors in the output, in particular whitespace errors within reason, and upper/lower case errors are often (but not always) ignored. But not printing any spaces at all (e.g. missing the space in the string “1 2” so that it becomes “12”) is typically not accepted. The safest way to get accepted is to follow the output format exactly.
- For problems with floating point output, we only require that your output is correct up to some error tolerance. For example, if the problem requires the output to be within either absolute or relative error of 10^{-4} , this means that
 - If the correct answer is 0.05, any answer between 0.0499 and .0501 will be accepted.
 - If the correct answer is 500, any answer between 499.95 and 500.05 will be accepted.

Any reasonable format for floating point numbers is acceptable. For instance, “17.000000”, “0.17e2”, and “17” are all acceptable ways of formatting the number 17. For the definition of reasonable, please use your common sense.

- An *interactive* problem is a problem where your program will be communicating back and forth with a judge program, e.g. playing a game against an opponent.

In these problems, you still read from standard input and write to standard output just as in other problems, but for these problems you need to also make sure to *flush* your output after sending a message to the judge program (otherwise it may just be placed in an internal buffer in your program and not sent). If you are using output streams in Java or C++ this can be done by calling the `.flush()` method of your output stream.

For interactive problems we sometimes also provide a simple testing tool program that you can use to test-run your solution. When available you will find this tool under “Download” to the right of the problem statement on the page for the problem in Kattis. Note that this tool is generally only meant to facilitate basic testing of your program, and that your solution might not necessarily be tested in the same way when submitted.

Problem A

Array of Discord

Time limit: 0 seconds

Once upon a time, high up on Mount Olympus, it came to pass that the gods held a competition to see who among them was the best at sorting lists of integers. Eris, the goddess of discord, finds this terribly boring and plans to add some mischief to the mix to make things more fun. She will sabotage the answers of Zeus so that his list of numbers is no longer sorted, which will no doubt be so embarrassing that he becomes furious and starts a minor war.



Eris clip art from freesvg, public domain

Eris must be careful not to be discovered while performing her sabotage, so she decides to only change a single digit in one of the numbers in Zeus' answer. The resulting number may not have any leading zeros (unless it becomes equal to zero in which case a single zero digit is allowed). Eris can only replace a digit with another digit – adding or removing digits is not allowed.

Input

The first line of input contains n ($2 \leq n \leq 100$), the length of Zeus' answer. The second line contains n integers a_1, a_2, \dots, a_n ($0 \leq a_1 \leq a_2 \leq \dots \leq a_n \leq 10^{15}$), Zeus' answer.

Output

If Eris can make the list not be sorted by changing a single digit of one of the numbers, then output n integers b_1, \dots, b_n , the resulting list of numbers after making the change. Otherwise, output "impossible". If there are many valid solutions, any one will be accepted.

Sample Input 1

```
3
2020 2020 2020
```

Sample Output 1

```
2021 2020 2020
```

Sample Input 2

```
2
1 99999999
```

Sample Output 2

```
impossible
```

Sample Input 3

```
4
1 42 4711 9876
```

Sample Output 3

```
1 42 4711 3876
```

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Problem B

Big Brother

Time limit: 0 seconds

You have come up with a new brilliant idea of automatically keeping track of how much (or little) your employees are working in the office: face recognition! By installing some advanced CCTV cameras in the office you will be able to automatically detect when the staff arrives or leaves, are taking breaks etc, thus reducing the need for manual administrative work. No more stamping clocks.



Picture by mi_brami, public domain

A good CCTV camera is expensive, so ideally you would only use one. It would obviously have to be placed somewhere where the entire office floor can be overlooked, so there are no walls blocking some dark corner of the floor where your workforce might hide.

While looking at the floor map, which can be modelled as a simple polygon, you are not sure if this is possible. Since the task is way above the paygrade of everyone else in the company you will have to write the program figuring this out yourself. If it is possible, you also want to know the area of the surface where the camera could be placed. See Figure B.1 for an example.

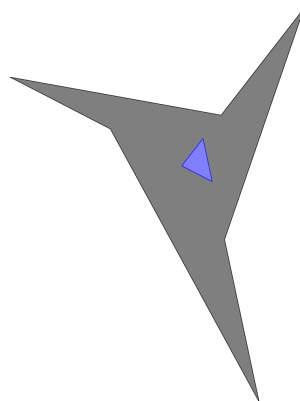


Figure B.1: Illustration of Sample Input 3. The blue shaded area in the middle indicates the region where the camera can be placed.

Input

The first line of input contains an integer n ($3 \leq n \leq 500\,000$), the number of vertices describing the polygon representing the office floor. Then follow n lines containing the integer coordinates x, y of the polygon in clockwise order ($0 \leq x, y \leq 10^7$).

Output

Output the area of the region of the map where a CCTV camera could be placed so that the rest of the office can be observed. (If it is not possible to put the camera anywhere, this area is 0.)

The answer must be correct with a relative of at most 10^{-6} , or an absolute error of at most 0.1.

Sample Input 1

```
8
0 0
0 1
1 1
1 2
2 2
2 1
3 1
3 0
```

Sample Output 1

```
1.0
```

Sample Input 2

```
8
0 0
0 2
1 2
1 1
2 1
2 2
3 2
3 0
```

Sample Output 2

```
0.0
```

Sample Input 3

```
6
140 62
97 141
68 156
129 145
153 176
130 109
```

Sample Output 3

```
48.80349500
```

Problem C

Coin Stacks

Time limit: 0 seconds

A and B are playing a collaborative game that involves n stacks of coins, numbered from 1 to n . Every round of the game, they select a nonempty stack each, but they are not allowed to choose the same stack. They then remove a coin from both the two selected stacks and then the next round begins.

The players win the game if they manage to remove all the coins. Is it possible for them to win the game, and if it is, how should they play?



Picture by KMR Photography via Wikimedia Commons, cc by

Input

The first line of input contains an integer n ($2 \leq n \leq 50$), the number of coin stacks. Then follows a line containing n nonnegative integers a_1, a_2, \dots, a_n , where a_i is the number of coins in the i 'th stack. The total number of coins is at most 1 000.

Output

If the players can win the game, output a line containing “yes”, followed by a description of the moves. Otherwise output a line containing “no”. When describing the moves, output one move per line, each move being described by two distinct integers a and b (between 1 and n) indicating that the players remove a coin from stacks a and b . If there are several possible solutions, output any one of them.

Sample Input 1

```
3
1 4 3
```

Sample Output 1

```
yes
1 2
2 3
2 3
2 3
```

Sample Input 2

```
3
1 1 1
```

Sample Output 2

```
no
```

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Problem D

Dams in Distress

Time limit: 0 seconds

Freyr, the god of prosperity, rain and the harvest, is having a lot of trouble these days. The giants are once again trying to invade Midgard, and have built a war camp at the bottom of the many valleys leading to Midgard. Now Freyr needs to wash that camp away, so a great victory feast can be held. Being at the bottom of the valley, any rain in the region can make its way through rivers and streams to the bottom of the valley and contribute to the glorious flooding of the giants. However, beavers and industrious humans have built dams throughout the river system, and these act as buffers that can hold some amount of water. But, on the flip side, once a dam is filled up to its capacity, it will break and all of the water stored there (as well as any further water added) will be released downstream.



Freyr by Johannes Gehrts (1901), Public Domain

Freyr, being the god of rain, knows exactly how much water is needed to wash the war camp away, and for each dam knows its exact capacity and how much water is currently stored there. Freyr, also being the god of prosperity and harvest, has better things to do than making it rain everywhere all day, so Freyr decides to only make it rain at a single place (either a dam, or the war camp), and to make it rain as little as possible in that place. What is the minimum amount of rain that Freyr needs to make to wash away the giants war camp, provided he carefully chooses the best location for the rain?

The network of dams and the war camp form a rooted tree, where the war camp is the root and the parent of a dam is the location (either another dam, or the war camp) immediately downstream of the dam. See Figure D.1 for an example.

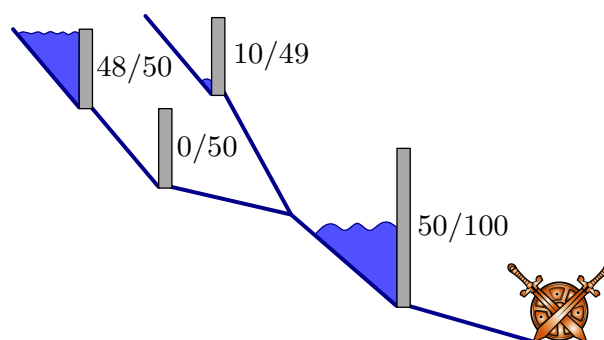


Figure D.1: Illustration of Sample Input 1. In this case Freyr only has to make 2 units of rain at the left-most dam in order to make it break and send 50 units of water downstream, which then ultimately results in 100 units of rain reaching the war camp, well exceeding the 75 units of water needed to flood the camp.

Input

The first line of input consists of two integers n and w ($1 \leq n \leq 2 \cdot 10^5$, $1 \leq w \leq 10^9$), the number of dams and the amount of water needed to wash away the war camp, respectively. Then follow n lines, describing the n dams. The dams are numbered from 1 to n .

The i th line contains three integers d_i , c_i and u_i ($0 \leq d_i < i$, $1 \leq c_i \leq 10^9$, $0 \leq u_i < c_i$), where d_i is the number of the dam immediately downstream of dam i (or 0 if the war camp is immediately downstream of dam i), c_i is the maximum capacity of dam i , and u_i is the current amount of water in dam i .

Output

Output the minimum amount of rain Freyr needs to make at one location, which will result in at least w water reaching the war camp.

Sample Input 1

```
4 75
0 100 50
1 49 10
1 50 0
3 50 48
```

Sample Output 1

```
2
```

Sample Input 2

```
4 13
0 12 1
1 6 1
2 4 1
3 10 0
```

Sample Output 2

```
10
```

Sample Input 3

```
4 1
0 100 50
1 49 10
1 50 0
3 50 48
```

Sample Output 3

```
1
```

Problem E

Exhaustive Experiment

Time limit: 0 seconds

You have been assigned to a new top-secret program involving a strange vacuum system. The physicists working on the system have been trying to find out where it is leaking but now they are confused by all the measurement results and want your help to figure out what is going on.

The vacuum system contains a wall with possibly leaking components. The physicists have performed vacuum leak tests on some of these components by flushing them with helium gas and then noting down whether their mass spectrometer detected any spike in helium in the vacuum system directly following this release of gas. If the component has even the tiniest leak they will detect it this way but there are some complications as well. The helium will rise up and spread out from where they released it and if it passes by any other leaking component, that will also trigger a positive reading. For each unit distance the helium has risen it will also have expanded by one unit. Thus the leak test will produce a positive result if the tested component is leaking or if there is a leaking component above it for which the x coordinates differs by at most half of the difference in the y coordinate. See Figure E.1 for an example.



Picture by Alf van Beem, public domain

You start out with a positive mindset thinking that there are probably just a few leaking components responsible for all the positive measurements. To determine if this is indeed possible you set out to determine the minimum number of leaking components that could give rise to the observed leak test results.

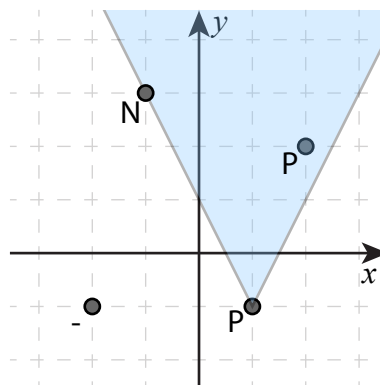


Figure E.1: Illustration of Sample Input 1. Circles indicate components and the blue triangle indicates where helium will spread when the first component is tested. This test being positive means that at least one of the the three components covered by the triangle is leaking. The correct answer in this case is 1 since the measurement results can all be explained with only the rightmost component leaking.

Input

The first line of input contains an integer n ($1 \leq n \leq 2 \cdot 10^5$), the number of components involved. The following n lines each contain two integers x and y and a character c ($-10^8 \leq x, y \leq 10^8$, $c \in \{-, P, N\}$), where (x, y) are the coordinates of a component and c describes a possible leak test result, with the following meanings:

- ‘-’ – No leak test has been performed on this component
- ‘N’ – Leak test gave negative response on this component
- ‘P’ – Leak test gave positive response on this component

No two components have the same position.

Output

Output a single integer, the minimum number of leaking components that could give rise to the observed leak test results. If no set of leaking components could give rise to the observed results, instead output the single word `impossible`.

Sample Input 1

```
4
1 -1 P
2 2 P
-1 3 N
-2 -1 -
```

Sample Output 1

```
1
```

Sample Input 2

```
2
0 0 N
1 2 P
```

Sample Output 2

```
impossible
```

Problem F

Film Critics

Time limit: 0 seconds

The premier of the anticipated action film *No Thyme to Fry* is right around the corner, and it is time to give early screenings to film critics so that they can review it. A small cinema has been selected to show these early screenings.



Picture by Mattias Ek, cc by

There are n critics numbered from 1 to n scheduled to watch the movie early, and each of them will watch it separately. After watching it, they will immediately give it a score from 0 to m . Susan, the cinema owner, has carefully looked at every critics social media and already knows that the i th critic thinks the movie is worth a score of a_i . However, the i th critic will not simply give the movie a score of a_i like you would expect, because they also take into account the scores that the other critics gave. Here is how they behave:

1. The first critic to arrive will be so happy that they are the first to review the movie that they will give it a score of m regardless of their initial opinion.
2. Every subsequent critic will look at the average score given by the previous critics. If this number is smaller than or equal to the initial opinion a_i then the critic will give it a score of m , otherwise they will give it a 0.

Susan thinks the critics behaviour is ridiculous. She has watched the movie, and it is clearly worth a score of exactly k/n and nothing else! But Susan is the owner of the cinema, so she gets to decide in what order to invite the critics. Your task is to find a permutation of $1, 2, \dots, n$ so that if the critics arrive in this order the average score will be exactly k/n .

Input

The first line of input contains three integers n , m and k ($1 \leq n \leq 2 \cdot 10^5$, $1 \leq m \leq 10^4$, $0 \leq k \leq n \cdot m$). The second line contains the n integers a_1, a_2, \dots, a_n ($0 \leq a_i \leq m$ for each i), the n critic scores as described above.

Output

If the critics can be ordered in such a way that the resulting average score is exactly k/n , then output n integers p_1, \dots, p_n , where $1 \leq p_i \leq n$ indicates that the i th critic to visit the cinema is the critic numbered p_i . This list of integers should be a permutation such that the average score given by the critics is k/n . If there are multiple solutions any one will be accepted.

Otherwise, if there is no such way to order the critics, output “impossible”.

Sample Input 1

```
5 10 30
10 5 3 1 3
```

Sample Output 1

```
3 5 2 1 4
```

Sample Input 2

```
5 5 20  
5 3 3 3 3
```

Sample Output 2

```
impossible
```

Problem G

Gig Combinatorics

Time limit: 0 seconds

Your friend Tóti is an aspiring musician. He has written n songs, each of which has *hype-rating* of either 1, 2, or 3. A higher hype-rating means the song has more energy. Tóti is planning his first live performance and needs your help. He wants to know how many *setlist* he can make. A setlist consist of at least three songs, the first songs has to have a hype-rating 1, the last has to have a hype-rating 3, and all others have to have a hype-rating 2. Tóti also wants to play the songs in the same order he wrote them.



Picture by Veress Attila Esküvői, cc by-sa

Given the hype-rating of each of Tóti's song in the order he wrote them, how many setlist can he make?

Input

The first line of the input consists of a single integer n ($1 \leq n \leq 10^6$), the number of songs Tóti has written. The second line consists of n integers, each in $\{1, 2, 3\}$, giving the hype-ratings of the n songs in the order they were written.

Output

The output should consist of one integer, the number of setlists Tóti can make. Since this number can be large, print it modulo $10^9 + 7$.

Sample Input 1

```
9
1 1 1 2 2 2 3 3 3
```

Sample Output 1

```
63
```

Sample Input 2

```
8
1 2 1 2 3 1 2 3
```

Sample Output 2

```
15
```

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Problem H

Hiring and Firing

Time limit: 0 seconds

Amazon' Inc, an up-and-coming company in e-commerce, has recently optimized its operations to make the most out of its workers. Thanks to state-of-the-art prediction methods, Amazon' now knows in advance how many workers will be needed each day for the foreseeable future. Using this information they can adjust the size of their workforce on a day-to-day basis by firing and/or hiring workers so that they always have exactly as many as are needed each day. In order to prevent the workers from getting too comfortable and organizing themselves, they will also regularly fire workers and replace them with new ones.



Image by Gerd Altmann via Pixabay

For instance, if on some day four more workers are needed than yesterday, Amazon' might fire 10 people and then hire 14 new ones on that day.

Unfortunately, due to labor laws, the firing of workers must follow a last-in-first-out order: the people who have been employed the shortest time must be fired first. Furthermore, a fired person cannot be re-hired within the foreseeable future so it is not possible to circumvent the law by firing some people and then immediately re-hiring some of them.

But this story is actually about HR, not workers! Every day, one employee from the HR department is assigned to be responsible for giving the fired workers the bad news that they are fired, and for then giving the newly hired workers the good news that they are hired. In order to minimize work environment problems in the form of social awkwardness for the HR staff, a policy has been established requiring that the HR person firing an employee must always be a different HR person than the one welcoming them when they were hired.

Now the time has come for the HR department to also optimize itself, by making itself as small as possible. Unlike workers, new HR staff cannot be hired with short notice, so the HR personnel must be permanent employees. What is the smallest number of HR people needed in order to manage all the planned hirings and firings?

Input

The first line of input contains an integer n ($1 \leq n \leq 10^5$), the length in days of the foreseeable future. Then follow n lines, the i th of which contains two integers f_i and h_i ($0 \leq f_i, h_i \leq 10^6$) where f_i is the number of workers fired on day i and h_i the number of people hired.

The number of workers fired on a day is never larger than the number of currently employed workers (in other words, $f_i \leq \sum_{j=0}^{i-1} h_j - f_j$ for all $1 \leq i \leq n$).

Output

Output a line with an integer k , the smallest number of HR people needed. The HR people are arbitrarily given IDs from 1 to k . Then output a line with n integers, the i th of which contains the ID of the HR person in charge of the firing and hiring on day i . If there is more than one solution, any one will be accepted.

Sample Input 1

```
4
0 3
1 1
2 1
2 0
```

Sample Output 1

```
3
1 2 3 2
```

Sample Input 2

```
6
0 10
0 5
2 0
0 0
0 100
50 100
```

Sample Output 2

```
2
1 2 1 2 1 2
```

Problem I

Infection Estimation

Time limit: 0 seconds

A new virus has appeared in country X, with a population of 10 million people. This time the country is prepared, and wants to start tracking its spread as quickly as possible. It is currently only known that at least 100 and at most 5 000 000 people are infected, and your job is to provide a more accurate estimate on the number of infected people.



Picture by Luca Volpi, cc by-sa

While it will take some time until tests get into mass production, one of the research labs is able to perform up to 50 tests per day. To improve test efficiency, the researchers have decided to combine tests from multiple people. Each test takes in tissue samples from any chosen number of people, and gets a positive result if there is virus in at least one of them, otherwise a negative result. The tests are performed sequentially – the result of each test becomes available before the next test can be performed.

Write a program which decides how to perform the tests and provides an estimate of the number of infected people which is within a factor 2 of the actual number of infected people.

Interaction

Your program can run up to 50 tests, and must then produce an estimate of the number of infected people. To issue a test, output “test k ” for an integer $1 \leq k \leq 10^7$. The judge will then provide a line which contains either “1” if the test for k randomly chosen people came back positive, or “0” if it came back negative. The k people will be chosen without replacement, i.e., the same person cannot be chosen twice. However, the tests are independent, so a person may end up being chosen in more than one round.

To provide the estimate, output “estimate x ”, where $0 \leq x \leq 10^7$ is your estimate on the number of infected people. The answer will be treated as correct if it is within a factor 2 of the correct answer y , i.e., if $y/2 \leq x \leq 2y$.

There will be a total of 100 runs of your program. You may assume that each run is deterministic: making the same sequence of tests on the i th run will always result in the same sequence of test results. To facilitate testing of your solutions, we provide a simple tool that you can download from the Kattis page for this problem. Remember to flush your standard output buffer for every line you output!

(Note: the sample interaction below is shown only for the purpose of illustrating the interaction protocol: there is no way the solution could reliably conclude the given estimate of 250 000 infected people based on the four tests performed.)

Read	Sample Interaction 1	Write
	test 10	
1		
	test 10	
0		
	test 17	
0		
	test 16	
1		
	estimate 250000	

Problem J

Joining Flows

Time limit: 0 seconds

Having recently taken over the Wonka Factory, Charlie is now in charge of the day-to-day production of the various chocolate products made there. While this may seem like a cushy job with an all-you-can-eat-chocolate perk, it also comes with the difficult responsibility of keeping the (somewhat convoluted and complicated) production lines working.



Picture by André Karwath, cc by-sa

The heart of the factory is the Chocolate River, where raw molten chocolate flows from k chocolate-producing faucets, to outlets where different types of pralines and chocolate bars are made. The i 'th of the k chocolate faucets produces chocolate at some fixed temperature t_i , and the amount of chocolate flowing from the faucet can be adjusted to any value between a_i and b_i millilitres per second. Suppose the k taps are adjusted to produce x_1, x_2, \dots, x_k millilitres of chocolates per second respectively (where $a_i \leq x_i \leq b_i$). Then the total flow in the Chocolate river is $x_1 + x_2 + \dots + x_k$, and its temperature is the weighted average

$$\frac{x_1 t_1 + x_2 t_2 + \dots + x_k t_k}{x_1 + x_2 + \dots + x_k}$$

(each faucet produces grade A quality chocolate which instantly mixes with the chocolate from the other faucets).

Each type of praline and chocolate bar produced at the factory requires the Chocolate River to be adjusted to have a specific temperature and flow level. Charlie recently came across a long list of new praline recipes, and would now like to figure out which of these are even possible to make at the factory. Write a program to determine, for each of the new recipes, if its required temperature and flow level is possible to achieve with some setting of the k faucets.

Input

The first line of input contains an integer k ($1 \leq k \leq 10$), the number of taps. Then follow k lines, describing the taps. The i 'th of these lines contains the three integers t_i , a_i , and b_i ($0 \leq t_i \leq 10^6$, $0 \leq a_i \leq b_i \leq 10^6$) describing the i 'th faucet.

Next follows a line containing an integer r ($1 \leq r \leq 10^5$), the number of new recipes to check. Then follows r lines, each describing a recipe. A recipe is described by two integers τ and ϕ ($0 \leq \tau \leq 10^6$ and $1 \leq \phi \leq 10^6$), where τ is the chocolate temperature and ϕ the chocolate flow needed for this recipe.

Output

For each of the r recipes, print one line with the string “yes” if it is possible to achieve the desired combination of chocolate temperature and flow, and “no” otherwise.

Sample Input 1**Sample Output 1**

```
2
50 0 100
100 50 100
3
20 75
75 150
75 90
```

```
no
yes
no
```

Problem K

Keep Calm And Carry Off

Time limit: 0 seconds

Petra is learning how to add two positive integers in school, but thinks it is a bit too difficult. They are currently working with the standard algorithm for addition, where you first compute the sum of the two units digits, then the sum of the two tens digits, and so on. Whenever the sum of the digits at the same position in the two numbers exceeds 9, a carry digit is added onto the digit of the next higher magnitude. Petra has trouble with the last step – she often forgets to keep track of the carry digit.

A few weeks ago, she also learnt a simpler method of addition. In this method, you repeatedly add 1 to one of the numbers and subtract 1 from the other, until the second one reaches zero. This can of course take a lot of time for large numbers.

Petra now wants to combine the two methods, for fast and error-free addition. Her plan is to first perform the second method one step at a time, until the two numbers would not produce a carry digit when added using the standard algorithm (for positive integers, this always happens eventually). To evaluate the performance of her new method, she has asked you to help her compute the number of steps she must perform of the second method when adding two given integers. Petra may perform the addition by 1 to either of the two numbers (and subtraction by 1 from the other).



Picture by Dennis Clarisse

Input

The input consists of two lines, each containing a positive integer with at most 10^6 digits. These are the two integers Petra wants to add.

Output

Output a single integer, the minimum number of times Petra must add 1 to one of her numbers (while subtracting 1 from the other) until they can be added using the standard addition algorithm without any carry digits.

Sample Input 1	Sample Output 1
10 99	1
Sample Input 2	Sample Output 2
90 10	10
Sample Input 3	Sample Output 3
23425 487915	12085

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Problem L

Language Survey

Time limit: 0 seconds

In Gridnavia, three languages are spoken: Arwegian, Banish, and Cwedish. Gridnavia consists of an $n \times m$ grid, where in each cell some non-empty subset of languages is spoken. It is known that each of the three languages is spoken in a non-empty connected subset of grid cells. Connected means that it is possible to get between any pair of cells by moving through adjacent cells, where two cells are said to be adjacent if they share a side.



Picture by LA2, cc by-sa

You have made a survey to find where in Gridnavia each language is spoken. The following question was sent to every cell in the region: “Please indicate if one or several of the languages is spoken in your cell”. But due to a misprint, there were no choices after the question, so everyone just wrote “one” or “several”.

So the only thing you know is for each cell whether exactly one, or more than one language is spoken in that cell.

To make the best out of the situation, you should find any division of the three languages that corresponds to the information.

Input

The first line of input contains two integers n and m ($1 \leq n, m \leq 200$).

The following n lines each contain a string of length m , consisting of the characters 1 and 2. The j th character on the i th line is 1 if exactly one language is spoken in that cell, and 2 if at least two languages are spoken.

Output

If the languages can be divided according to the information, then output three copies of the grid. The first copy should consist of characters “A” and “.”, where the “A” indicates that Arwegian is spoken in that cell, and “.” indicates that it isn’t spoken. The following two grids should consist of characters “B” and “.”, and “C” and “.”, respectively, with the corresponding information about Banish and Cwedish.

Remember that the three regions have to be connected and non-empty, and every cell must be part of some region. For readability, it is recommended to put empty lines in between the three grids, but it is not necessary to do so. If there are multiple solutions any one will be accepted.

Otherwise, if there is no way to divide the languages, output “impossible”.

Sample Input 1

```
3 4
2211
1112
1112
```

Sample Output 1

```
AAAA
...A
....

BB..
BBBB
...B

....
...C
CCCC
```

Sample Input 2

```
1 1
1
```

Sample Output 2

```
impossible
```

Problem M

Methodic Multiplication

Time limit: 0 seconds

After one computer crash too many, Alonso has had enough of all this shoddy software and poorly written code! He decides that in order for this situation to improve, the glass house that is modern programming needs to be torn down and rebuilt from scratch using only completely formal axiomatic reasoning. As one of the first steps, he decides to implement arithmetic with natural numbers using the Peano axioms.



Giuseppe Peano, public domain

The Peano axioms (named after Italian mathematician Giuseppe Peano) are an axiomatic formalization of the arithmetic properties of the natural numbers. We have two symbols: the constant 0, and a unary successor function S . The natural numbers, starting at 0, are then 0, $S(0)$, $S(S(0))$, $S(S(S(0)))$, and so on. With these two symbols, the operations of *addition* and *multiplication* are defined inductively by the following axioms: for any natural numbers x and y , we have

$$\begin{array}{ll} x + 0 = x & x \cdot 0 = 0 \\ x + S(y) = S(x + y) & x \cdot S(y) = x \cdot y + x \end{array}$$

The two axioms on the left define addition, and the two on the right define multiplication.

For instance, given $x = S(S(0))$ and $y = S(0)$ we can repeatedly apply these axioms to derive

$$\begin{aligned} x \cdot y &= S(S(0)) \cdot S(0) = S(S(0)) \cdot 0 + S(S(0)) \\ &= 0 + S(S(0)) = S(0 + S(0)) = S(S(0 + 0)) = S(S(0)) \end{aligned}$$

Write a program which given two natural numbers x and y , defined in Peano arithmetic, computes the product $x \cdot y$.

Input

The input consists of two lines. Each line contains a natural number defined in Peano arithmetic, using at most 1 000 characters.

Output

Output the product of the two input numbers.

Sample Input 1

```
S ( S ( 0 ) )
S ( S ( S ( 0 ) ) )
```

Sample Output 1

```
S ( S ( S ( S ( S ( S ( 0 ) ) ) ) ) ) )
```

Sample Input 2

```
S ( S ( S ( S ( S ( 0 ) ) ) ) ) )
0
```

Sample Output 2

```
0
```

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